Robustness – Theoretical framework

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- Introduction
- Eurocodes - Robustness
- Robustness - Framework and indicators
- Summary

Introduction

Why is robustness an issue?

Oklahoma bombing
1995

World Trade Center
2001

Charles de Gaulle
2004

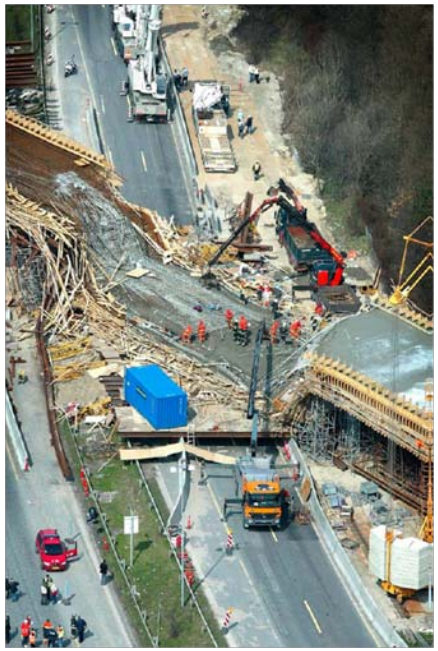
Introduction

Reasons to failures:

- Extreme high load / extreme low strength: very unlikely (probability of failure per year ~ 10^{-5} - 10^{-6})

  covered by 'component-based' design rules and psf in codes

- Other reasons:
  - Design errors
  - Execution errors
  - Deterioration of critical structural elements / lack of maintenance
  - Unexpected hazards - unforeseeable incidents
  - System effects

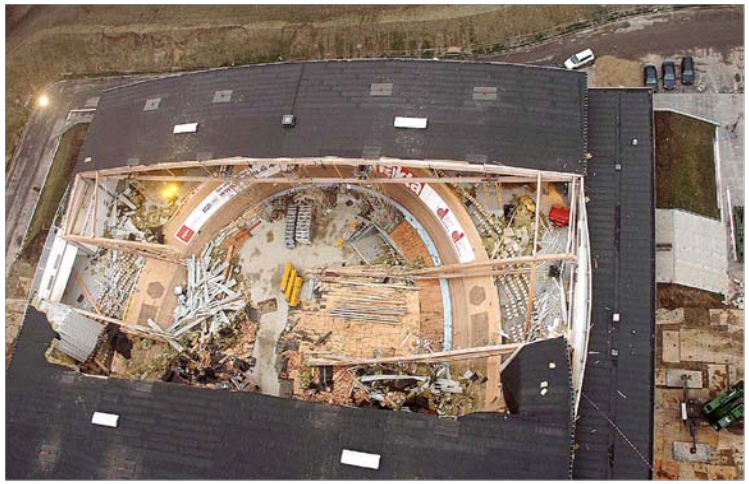


→ (to be) covered (partly) by 'Robustness requirements' in codes

Introduction

Ballerup arena - 2003
Copenhagen, Denmark

2 out of 12 main trusses collapsed



Ice skating arena - 2006
Bad Reichenhall, Germany

Total collapse

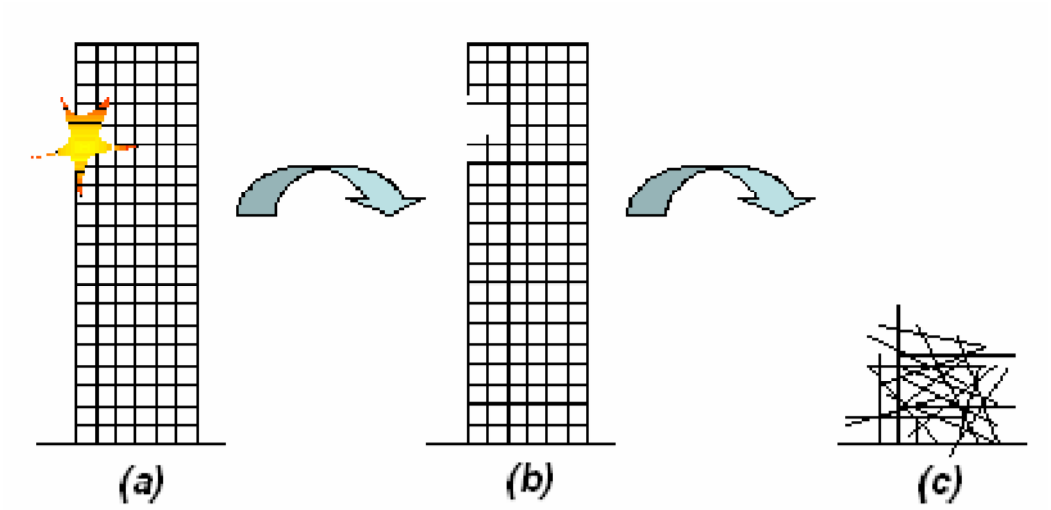
- Hazards: design error, unforeseen incidents, …
  - Correlated / uncorrelated for different elements?
  - New / conventional system?
- Connection between main trusses/beams: strong / weak?
  - Series / parallel (redundant) system?
- Brittle / ductile failure type?

Robustness - Eurocodes

EN1990 and EN1991-1-7

A structure shall be designed and executed in such a way that it will not be damaged by events such as :

- explosion,
- impact, and
- the consequences of human errors, to an extent disproportionate to the original cause.

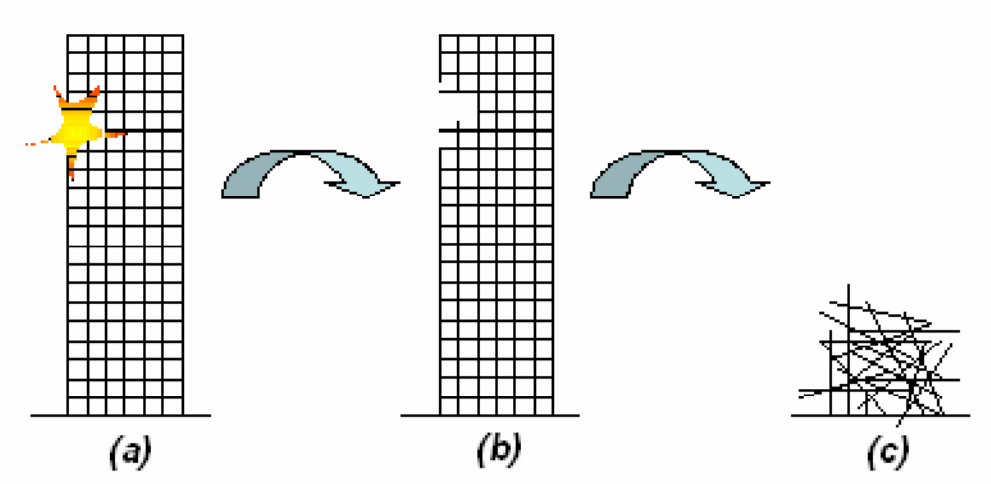


Robustness - Eurocodes

Potential damage shall be avoided or limited by:

- avoiding, eliminating or reducing the hazards to which the structure can be subjected
- selecting a structural form which has low sensitivity to the hazards considered
- selecting a structural form and design that can survive adequately the accidental removal of an individual member or a limited part of the structure, or the occurrence of acceptable localised damage
- avoiding as far as possible structural systems that can collapse without warning → *(ductility)*
- tying the structural members together

Robustness – probabilistic model



- Exposure - E_i: $P(EX_i)$
- Damage due to exposure - D_j : $P(D_j|EX_i)$
- Consequence – Collapse: $P(\text{Collapse}|EX_i \cap D_j)$

Total probability of collapse:

$$P(\text{Collapse}) = \sum_i \sum_j P(\text{Collapse}|EX_i \cap D_j) P(D_j|EX_i) P(EX_i)$$

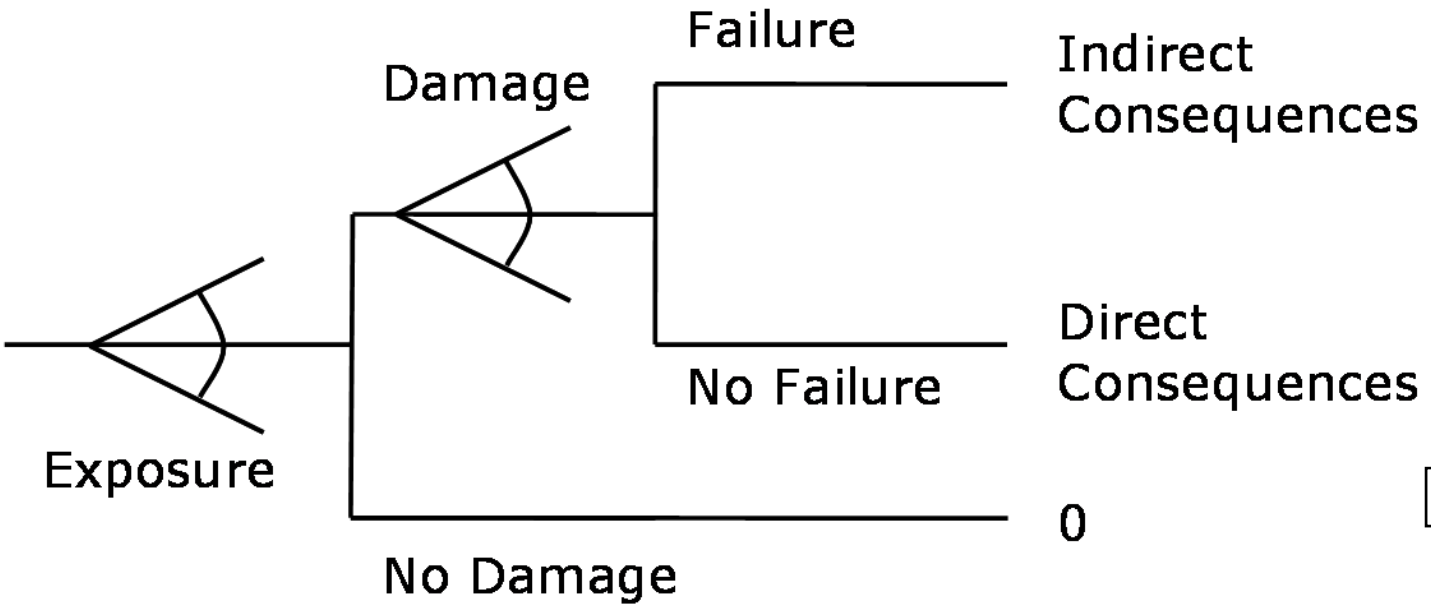
Robustness – probabilistic model

- Exposure - E_i: $P(EX_i)$
  - Examples:
    unforeseen load effects
    unforeseen settlements
    incorrect structural modelling
- Damage due to exposure - D_j : $P(D_j|EX_i)$
  - Examples:
    loss of column
    failure of part of storey area
- Consequence – Collapse $P(\text{Collapse}|EX_i \cap D_j)$
  - Example:
    collapse of major part of structural system

Total probability of collapse:

$$P(\text{Collapse}) = \sum_i \sum_j P(\text{Collapse}|EX_i \cap D_j) P(D_j|EX_i) P(EX_i)$$

Robustness – risk-based model

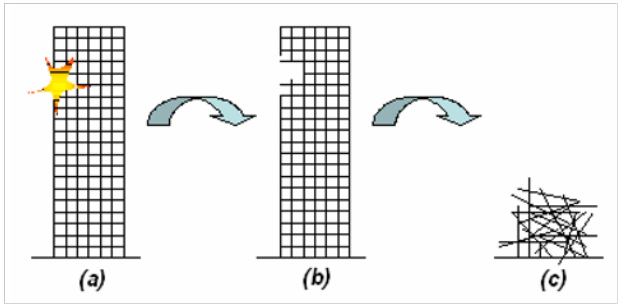


Baker et al. 2008

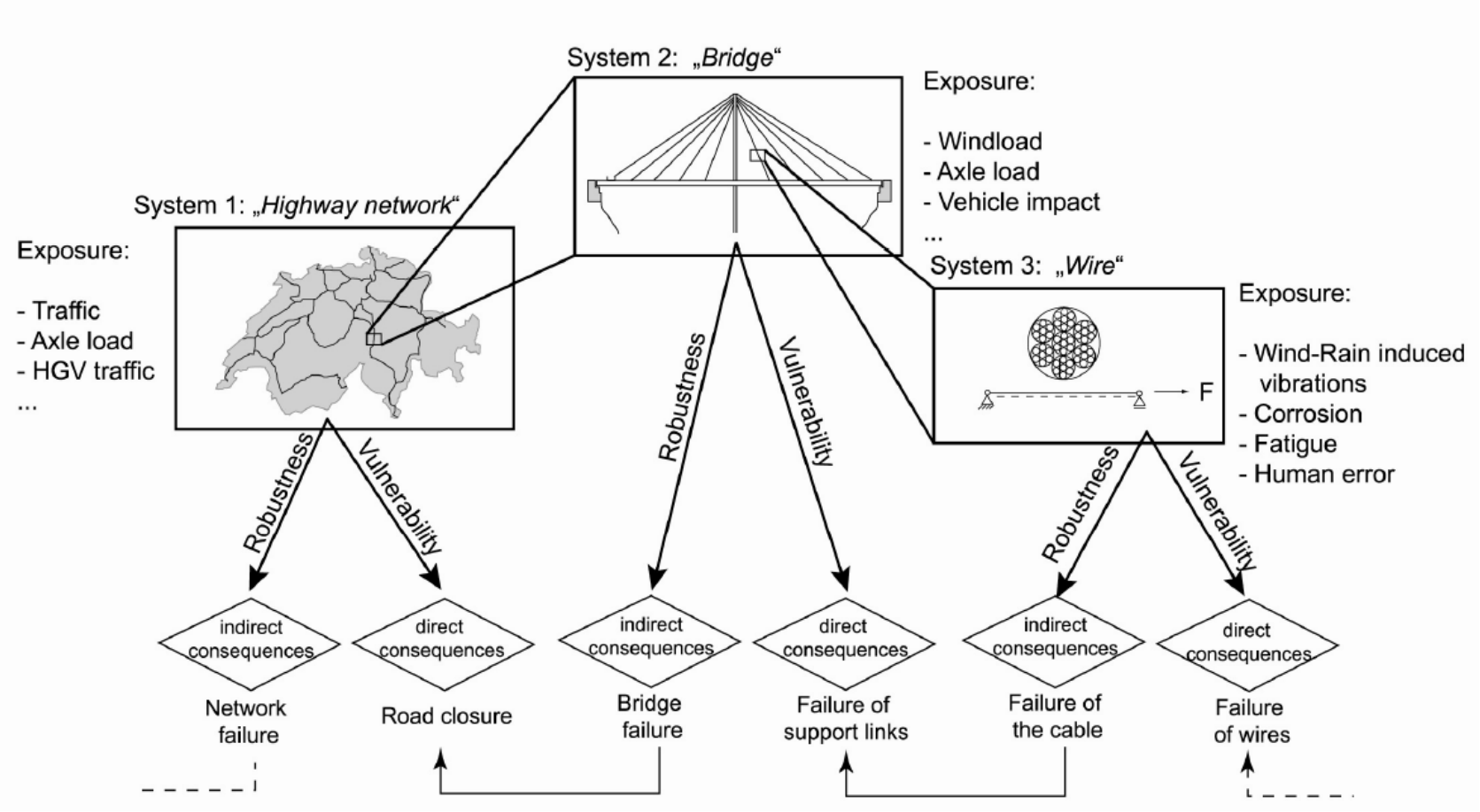
Total Risk = Direct Risk + Indirect Risk:

$$R = \sum_i \sum_j C_{\text{dir},ij} P\left(D_j \middle| EX_i\right) P\left(EX_i\right) +$$

$$\sum_k \sum_i \sum_j C_{\text{ind},ijk} P\left(S_k \middle| D_j \cap EX_i\right) P\left(D_j \middle| EX_i\right) P\left(EX_i\right)$$



Risk analysis – system modeling



JCSS 2008. Risk Assessment in Engineering Principles, System Representation & Risk Criteria.

Robustness - Indicators

- Risk-based robustness index

$$I_{rob} = \frac{R_{Dir}}{R_{Dir} + R_{Ind}}$$

high robustness : $I_{rob} \to 1$

low robustness : $I_{rob} \to 0$

- Reliability-based robustness indices

$$\beta_R = \frac{\beta_{\text{intact}}}{\beta_{\text{intact}} - \beta_{\text{damaged}}}$$

high robustness : $\beta_R \to \infty$

low robustness : $\beta_R \to 0$

$$RI = \frac{P_{f\,(\text{damaged})} - P_{f\,(\text{intact})}}{P_{f\,(\text{intact})}}$$

high robustness : $RI \to 0$

low robustness : $RI \to \infty$

- Deterministic robustness index, e.g. based on a pushover analysis

$$RIF_i = \frac{RSR_{\text{damaged}}}{RSR_{\text{intact}}}$$

high robustness : $R_i \to 1$

low robustness : $R_i \to 0$

Robustness - Indicators

Conditional risk-based robustness indicator:

$$I_{rob|\text{exposure/damage}} = \frac{R_{Dir|\text{exposure/damage}}}{R_{Dir|\text{exposure/damage}} + R_{Ind|\text{exposure/damage}}}$$

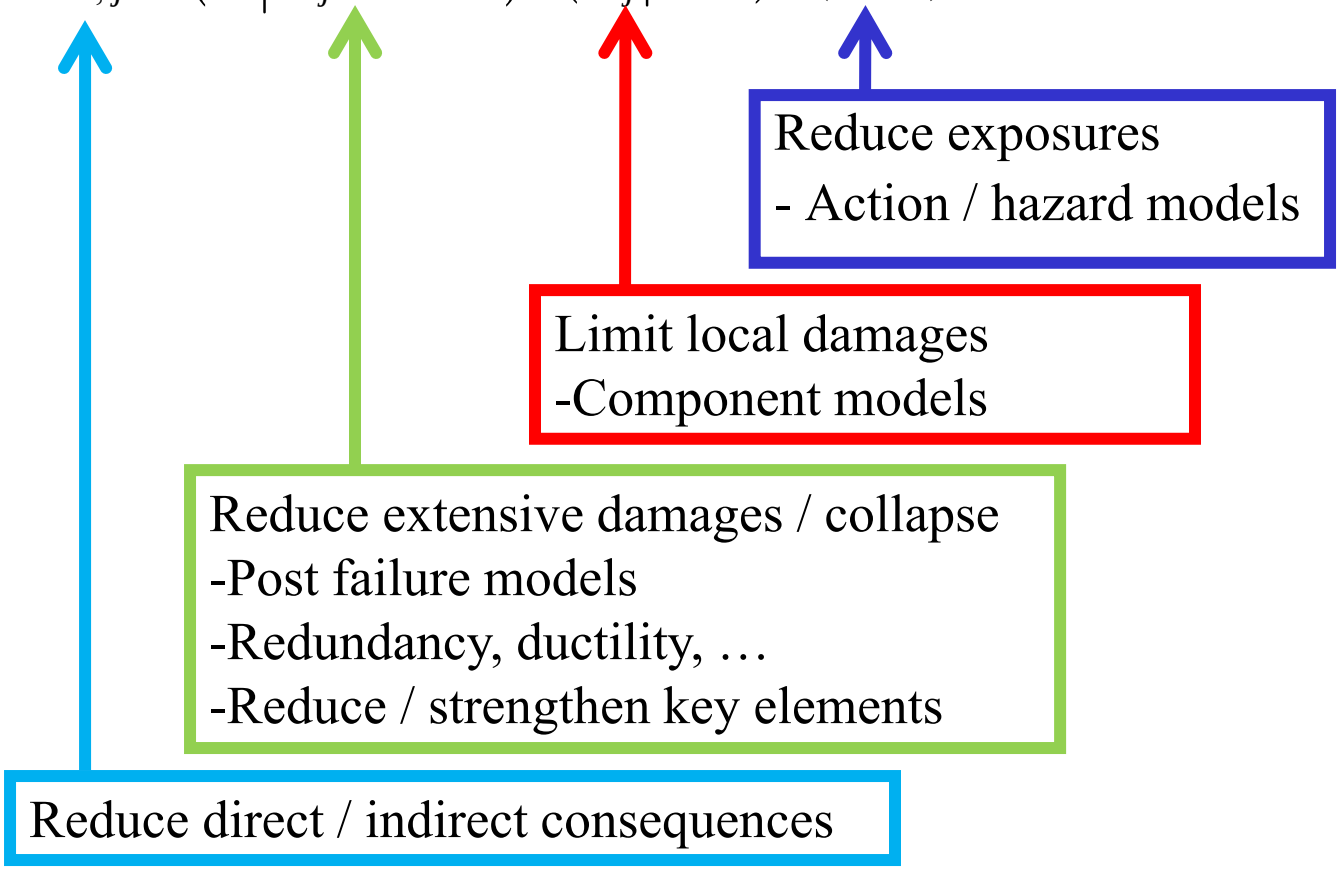
- conditional on given exposure and/or damage

Robustness

How to decrease risk / increase robustness?

$$\text{Risk} = \sum_i \sum_j C_{\text{dir},ij} P\left(D_j \middle| EX_i\right) P\left(EX_i\right)$$

$$+ \sum_k \sum_i \sum_j C_{\text{ind},ijk} P\left(S_k \middle| D_j \cap EX_i\right) P\left(D_j \middle| EX_i\right) P\left(EX_i\right)$$



Robustness

Potential damage shall be avoided or limited by:

- Avoiding, eliminating or reducing hazards
- Structural design with low sensitivity to hazards
- Structural design that can survive adequately the accidental removal of an individual member or limited part of the structure
- Avoiding structural systems that can collapse without warning → (ductility)
- Tying structural members together

- Requirements depend on consequence class (CC1, CC2 or CC3)

Robustness

- Not always a good idea to use redundant systems / tie elements together – use of statically determinate (series) systems can be better than a redundant system

Robustness strategy depend on

- Exposure type: design error, unforeseen incidents, …
- Correlation of exposure between elements
- New / conventional structural system
- Load bearing capacity: time dependency
- Load type: permanent / variable load dominating

Robustness - Codes

Code based design

Standard Code Format – Component based

- Safety format
- Design equations
- Enveloping loads
- Load combinations
- Material characteristics
- Characteristic values / partial safety factors / load combination factors
- etc.

Robustness requirements – System based

Quality control requirements - human errors
Inspection & maintenance - deterioration

Fact sheets - Robustness

1. Robustness – theoretical framework:
2. Robustness – acceptance criteria

3. Earthquake and robustness for timber structures
4. System reliability – ductility and redundancy
5. Robustness design of timber structures
6. Robustness ‘experiences’ from failed timber structures